THE ASTROPHYSICAL JOURNAL, 214:809–811, 1977 June 15

NONRADIAL PULSATIONAL INSTABILITY OF MASSIVE STARS

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Received 1976 June 8

ABSTRACT

Pulsational eigenfrequencies and stability coefficients of the lowest modes of nonradial quadrupole ($l = 2$) oscillation have been obtained for chemically homogeneous stellar models constructed with Carson's new radiative opacities. Instability is found to develop as a result of the same operation of the κ-mechanism in the CNO ionization zone and at nearly the same stellar masses as was the case for the radial ($l = 0$) modes studied previously. A survey of the $l = 5$ modes indicates that the instability gradually declines with increasing degree l of the spherical harmonics.

Subject headings: stars: interiors — stars: massive — stars: pulsation

I. INTRODUCTION

A large opacity in the CNO ionization zone of a star can lead to radial pulsational instability if the star's structure is such that this zone does not lie too close to or too far below the surface and if the star's mass is sufficiently large (Stothers 1976). For such a star, nuclear energy release inside the core is unimportant for driving radial pulsations because the temperature fluctuations are very small near the center. In the case of nonradial modes nuclear energy release inside the core is unimportant *in general* since the temperature fluctuations for these modes always go to zero at the center (Simon 1957; Wan Fook Sun 1966; Aizenman, Hansen, and Ross 1975). Thus it is of interest to see whether the opacity mechanism can lead to any instability toward nonradial pulsations in models of massive stars constructed with large CNO opacities.

II. ASSUMPTIONS

As in the earlier work on radial pulsations (Stothers 1976), a linear nonadiabatic perturbation analysis has been performed to obtain the eigenvalues, $\omega^2 = (2\pi/\text{Period})^2 R^3/GM$, and the stability coefficients for the lowest modes of nonradial pulsation of degree l, corresponding to the spherical harmonics $Y_l^m(\theta, \phi)$ in which the Eulerian perturbations of the physical variables are expanded. A reasonably good approximation for stellar models (such as the present ones) with a large effective polytropic index is to ignore the Eulerian perturbation of gravitational potential and the contribution to the stability integral from horizontal energy exchanges in the outer, nonadiabatic part of the star. The resulting set of nonradial pulsation equations, if reexpressed in a form employing Lagrangian perturbations, resembles the more familiar set of *radial* pulsation equations, but with the addition of a few extra terms. This approximate set of equations has been adopted here. Interaction between the pulsations and convection, which occurs both in the core and in the CNO ionization zone, has been ignored.

Further details of the input physics, method of solution of the equations, and equilibrium properties of the models are given in the earlier paper on radial pulsations (Stothers 1976). Two classes of nonrotating, chemically homogeneous models have been investigated in the present paper: hydrogen-burning models with $(X, Z) = (0.73, 0.02)$ and helium-burning models with $Z = 0.02$ and $Z = 0.04$. These models are all based on Carson's new radiative opacities, which are very large in the CNO ionization zone.

III. NONRADIAL PULSATIONAL PROPERTIES OF THE MODELS

Pulsational eigenvalues are listed in Tables 1 and 2 for a few selected hydrogen-burning and helium-burning models. Both radial ($l = 0$) and quadrupole ($l = 2$) nonradial modes are listed to afford an approximate comparison. (A precise comparison is not possible, because we have ignored the Eulerian perturbation of the gravitational potential in the nonradial pulsation equations.) Identification of the nonradial modes is based on Osaki's (1975) method, and the notation follows Cowling's (1941) convention. Radial modes are simply p modes of $l = 0$, but we have adopted the standard notation of 0 (fundamental mode), 1 (first overtone), and 2 (second overtone). Actually, in the case of stellar models with a low central condensation, both the eigenvalues and the number of nodes in the eigenfunctions for the radial modes 0, 1, 2, etc., resemble more closely those for the respective nonradial modes f, p_1, p_2, etc.

In general, as one allows the stellar mass to increase, the increasing radiation pressure inside the star reduces the effective ratio of specific heats, and the increasing stellar radius raises the central condensation. In the present models, the radial pulsation modes turn out to be affected more by the first factor, and so their eigenfrequencies drop; whereas the nonradial pulsation modes (especially the g^+ modes) are affected more by the second factor, and consequently their eigenfrequencies increase.

TABLE 1

PULSATIONAL EIGENFREQUENCIES (ω^2) OF THE HYDROGEN-BURNING MODELS WITH $(X, Z) = (0.73, 0.02)$

	$M/M_\odot$ AND $\log \rho_c/\langle\rho\rangle$						
MODE	30, 1.71	60, 1.98	120, 2.42	200, 2.66	400, 2.84	600, 2.87	1000, 2.82
				$l = 0$			
0	9.76	8.58	5.98	5.39	5.15	5.10	5.15
1	13.3	13.1	16.1	14.6	13.8	13.7	13.7
2	22.2	22.0	27.7	22.9	20.9	21.7	24.4
				$l = 2$			
g_2^+	0.85	1.23	2.89	4.02	4.53	4.54	3.88
g_1^+	1.75	1.96	3.96	5.23	5.69	5.33	4.95
f	12.5	8.91	7.45	9.09	10.8	11.0	10.4
p_1	16.0	21.1	17.2	16.8	17.3	17.9	17.4
p_2	24.8	26.9	28.0	23.6	22.2	23.0	25.4

In earlier work on nonradial eigenfrequencies of models of very massive stars, the opacity was taken to be purely electron scattering (Smeyers 1963, 1967; Van der Borght and Wan Fook Sun 1965) or electron scattering plus Kramers's law of opacity (Aizenman, Hansen, and Ross 1975). Combining these results with the present results, we find a monotonic increase of ω^2 with the relative contribution of atomic absorption to the total opacity. This trend is due to the change of effective polytropic index in the outer (radiative) part of the envelope, where $n_{\rm eff} = (3 + \eta)/(1 + \alpha)$, with $\alpha = \partial \ln \kappa/\partial \ln \rho$ and $\eta = -\partial \ln \kappa/\partial \ln T$. For an

TABLE 2

PULSATIONAL EIGENFREQUENCIES (ω^2) OF THE HELIUM-BURNING MODELS

A.

	$M/M_\odot$ AND $\log \rho_c/\langle\rho\rangle$ for $Z = 0.02$						
MODE	2, 1.52	3, 1.50	4, 1.56	6, 1.91	8, 2.31	10, 2.64	15, 3.22
				$l = 0$			
0	9.38	8.37	8.22	6.19	3.36	2.70	1.85
1	17.2	17.8	20.1	19.3	16.7	14.1	10.9
2	28.7	29.0	27.3	27.3	38.7	33.3	23.8
				$l = 2$			
g_2^+	0.92	0.72	0.68	1.15	2.37	4.14	9.81
g_1^+	2.09	1.70	1.62	2.78	5.73	8.37	10.6
f	10.2	10.0	11.2	7.04	6.23	10.2	20.4
p_1	18.7	19.0	21.8	24.9	19.2	18.6	27.8
p_2	29.8	30.6	27.5	27.6	42.9	36.4	36.1

B.

	$M/M_\odot$ AND $\log \rho_c/\langle\rho\rangle$ FOR $Z = 0.04$						
MODE	2, 1.58	3, 1.61	4, 1.79	6, 2.34	8, 2.84	10, 3.25	15, 4.14
				$l = 0$			
0	10.3	10.4	10.3	3.72	2.54	2.01	0.91
1	18.7	21.0	14.4	16.7	13.0	9.72	4.84
2	29.8	27.0	32.0	38.9	27.2	20.9	16.0
				$l = 2$			
g_2^+	1.11	0.95	1.18	3.28	8.08	9.39	37.1
g_1^+	2.51	2.23	2.83	6.20	9.25	17.1	50.7
f	11.0	12.2	10.8	7.93	18.5	18.2	58.5
p_1	19.5	22.4	18.4	18.4	19.6	34.5	70.4
p_2	30.3	27.9	34.5	40.1	31.1	42.1	88.8

electron-scattering opacity, $n_{\rm eff} = 3$; while, for Kramers's law of opacity, $n_{\rm eff} = 3.25$. Very crudely, for Carson's irregular opacities, $n_{\rm eff} \approx 3.5$. Robe's (1968) study of polytropes with different indices n shows explicitly the expected correlation between ω^2 and n for the lowest nonradial pulsation modes. A similar result holds for the radial pulsation modes, of course.

In the present series of stellar models, the stability coefficients for the lowest nonradial modes show approximately the same behavior with increasing stellar mass as they do for the radial modes studied earlier. Thus, in the case of the hydrogen-burning models, the critical mass dividing pulsationally stable and unstable models is apparently greater than 400 $M_\odot$, given the adopted input physics; but this value is very uncertain because of the complicating presence of convection in the envelope.

The critical mass for the helium-burning models, on the other hand, is much more accurately determined. Results for the lowest modes are given in Table 3. Additional calculations for the most unstable of the helium-burning models show that instability continues to much higher modes than those actually listed in the table. Stability coefficients have also been computed for the case $l = 5$. Comparison with the results for $l = 0$ and $l = 2$ indicates that the critical mass for any mode is nearly independent of l, i.e., it is about 4 $M_\odot$ ($Z = 0.02$) or 2 $M_\odot$ ($Z = 0.04$) for the fundamental (lowest) mode. However, pulsational stability is eventually recovered at very high stellar masses because of (1) the growing importance of convection in the envelope and (2) the decreasing surface temperature of the star. The recovery is more rapid if l is larger. Crude estimates yield a recovery of stability at 30, 25, and 12 $M_\odot$ for $l = 0$, 2, and 5 (with $Z = 0.02$); and at 15, 12, and 8 $M_\odot$ for $l = 0$, 2, and 5 (with $Z = 0.04$). These estimates apply to all the modes that we have considered except the fundamental radial mode, which remains unstable to indeterminately high masses.

TABLE 3

CRITICAL MASSES FOR PULSATIONAL INSTABILITY OF THE HELIUM-BURNING MODELS WITH NONADAPTIVE CONVECTION

MODE	CRITICAL $M/M_\odot$ $Z = 0.02$	CRITICAL $M/M_\odot$ $Z = 0.04$
	$l = 0$	
0.........	4	2
1.........	4	2
2.........	6	3
	$l = 2$	
g_2^+.......	7	5
g_1^+.......	6	4
f.........	4	2
p_1........	4	3
p_2........	6	3

IV. CONCLUSION

Nonradial pulsation modes in homogeneous stellar models constructed with Carson's opacities have stability characteristics that are very similar to those for the radial pulsation modes. Of particular interest are the many excited modes (of all degrees l that have been examined) in the massive helium-burning models. The periods of pulsation range from minutes to hours, and the e-folding times for the growth of the unstable modes are extremely short compared to the Kelvin time. These results are of obvious interest in connection with the instability of Wolf-Rayet stars.

Although the main mechanism inducing pulsational instability is the large opacity bump in the CNO ionization zone, we have also found that, in some cases, the g_3^+ and higher g^+ modes (with $\omega^2 < 1$) are destabilized at stellar masses well *below* the critical mass for the other modes (in both the helium-burning and the hydrogen-burning models) as a result of a secondary helium bump that shows up in Carson's opacities when Z is small, at temperatures slightly above 10^6 K, and at densities around 10^{-3} g cm^{-3}. Whether excited g^+ modes with very long periods can resonantly couple to lower modes with periods that are much shorter, and thereby account for the rapid variability of β Cephei stars, is an interesting question. However, since the secondary helium bump has not yet been fully confirmed by Carson, further speculation about its pulsational consequences seems to be unwarranted at the present time.

We thank Dr. T. Richard Carson for making available to us his unpublished opacities.

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